

# Universality of the Gutenberg-Richter Relationship *(with $b=1$ )*

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# Definition of Universal Gutenberg-Richter

An earthquake nucleating anywhere, at any time, will randomly grow to a magnitude  $\geq M$  according to the distribution

$$\log(N) = a - bM,$$

$\Rightarrow$  The probability that any nucleating earthquake will grow to magnitude  $\geq M$  is

$$\Rightarrow P(M) = 10^{b(M_{min} - M)}$$

$N = \# \text{ eqs } \geq M$ ,  $a \propto$  earthquake rate,  $b = \text{constant (1.0)}$ ,  
 $M_{min} = \text{minimum earthquake magnitude}$

# Definition of Universal Gutenberg-Richter

On a major fault ~



$$P(M) = 10^{b(M_{min} - M)}$$

In the boonies ~

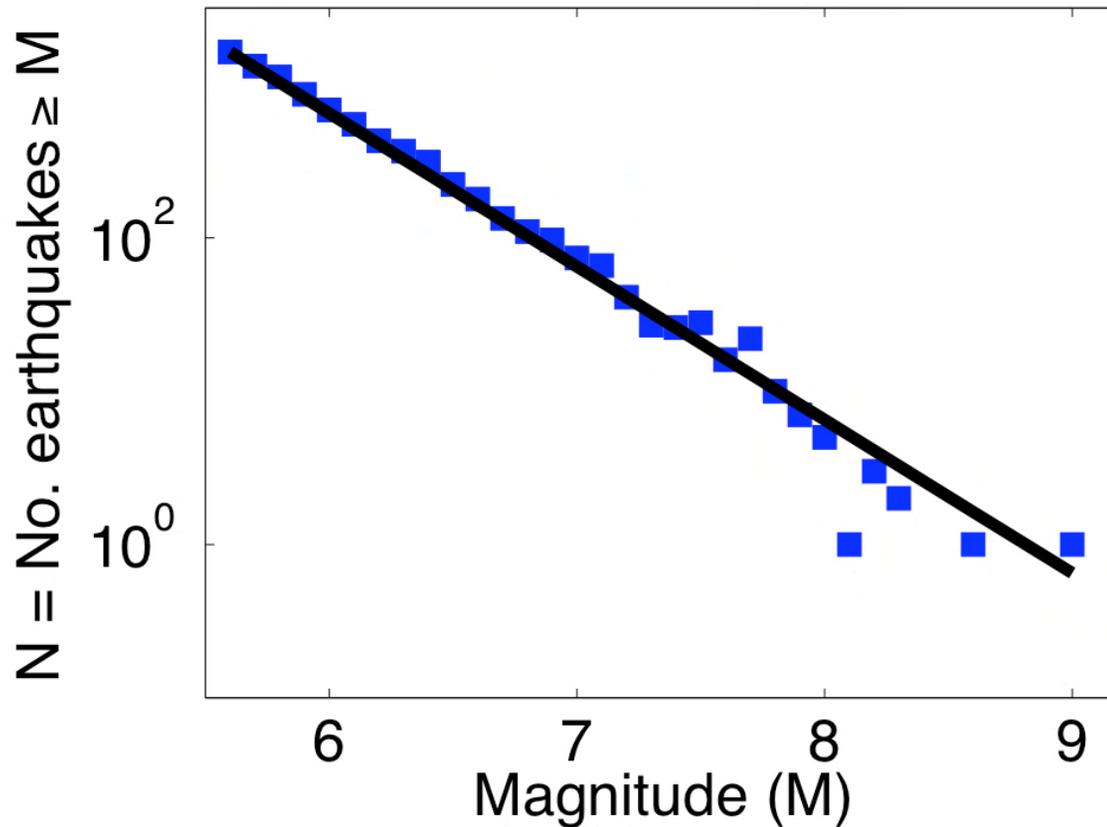


$$P(M) = 10^{b(M_{min} - M)}$$

(relationship applicable until region-specific  $M_{max}$ )

If the magnitudes associated with hypocenters occurring in any random box are sampled for long enough, the distribution will look like:

### 1976-2005 Global CMT catalog



# Why do we care if the GR relationship is universal?

- Big issue in earthquake hazard mapping
- Important implications for physics of earthquake growth, faults, and predictability

# Outline

1. A physical hypothesis for where the GR relationship comes from and why it should be universal
2. Traditional arguments for non-universal GR and where they go wrong

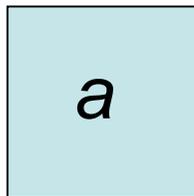
# Physical hypothesis for the GR relationship

The process of earthquake growth after nucleation may be modeled as a struggle between **pushing forces** (rupture-induced stresses) and **stopping forces** (resistance from the unruptured fault)

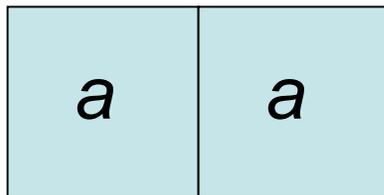


**We will show:** The GR relationship results if the pushing force exerted is linearly proportional to the current rupture area, while the stopping forces remain a constant

# Illustration of pushing force $\propto$ to current faulting area



A rupture that has an area of  $a$  has a 50% probability of growing to area  $2a$

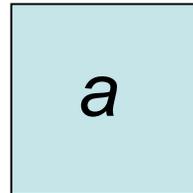


A rupture that has grown to  $2a$  has twice as much pushing power, so it has a 50% chance of growing to area  $4a$ .

And so on

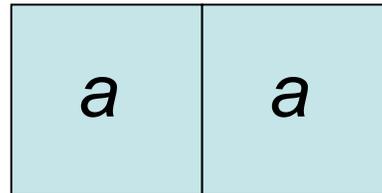
# Resulting area-frequency statistics

For every  $n$  ruptures  
of area  $a$



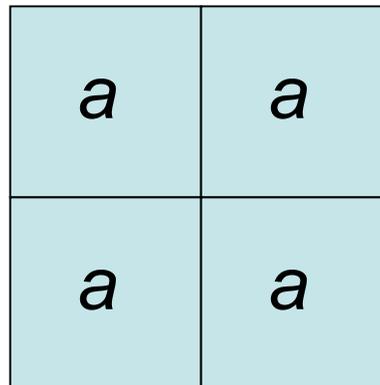
50% of these  
grow to area  $2a$

$\Rightarrow n/2$  ruptures of  
area  $2a$



50% of these  
grow to area  $4a$

$\Rightarrow n/4$  ruptures of  
area  $4a$



In general:  
 $N \propto 1/A$   
 $A = \text{total area}$

From  $N \propto 1/A$  we follow *Kanamori and Hanks (1975)* to get back to the GR relationship:

$$(1) \quad N \propto \frac{1}{A}$$

$$(2) \quad A \propto 10^M$$

$$(3) \quad N \propto 10^{-M}$$

Where (3) is the Gutenberg-Richter relationship with  $b=1$

Analogy: Push from current faulting area  
<=> Pull from a team of horses



2 horses have a probability  $P$  of making it  $X$  km

Resistance on carriage from road = constant



At  $X$  km, 2 more horses are added. They can now travel  $2X$  km with probability  $P$

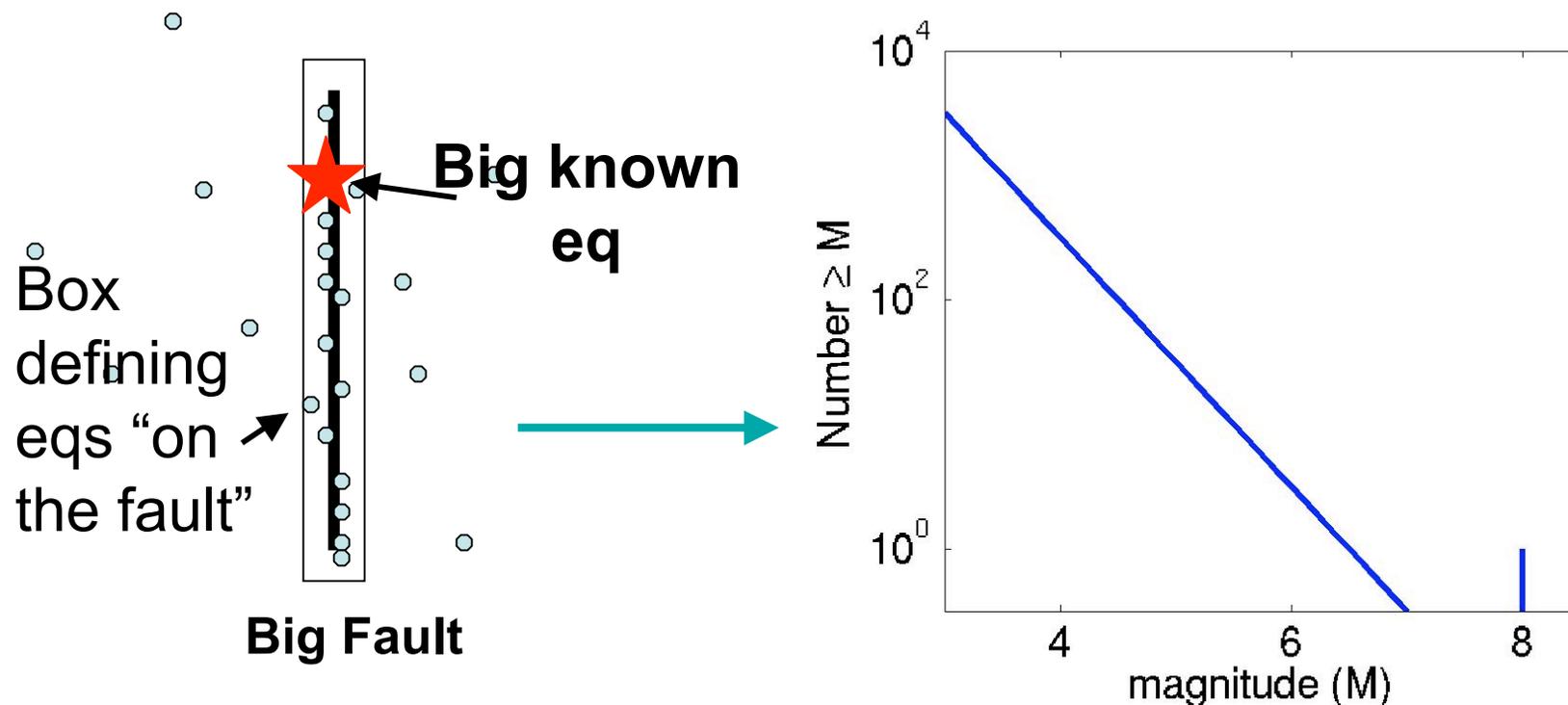
# How the strength of the horses and the smoothness of the cobblestones affect the results

- Smoother cobblestones (**smoother/more developed fault**) does not change the relative distance that 2 vs. 4 horses can cover
  - The strength of the horses (**stress on the fault**) does not change the relative distance that the horse teams can cover
- => The GR magnitude distribution is universal -- not affected by variations in physical conditions**

# Arguments for non-universal GR and their problems

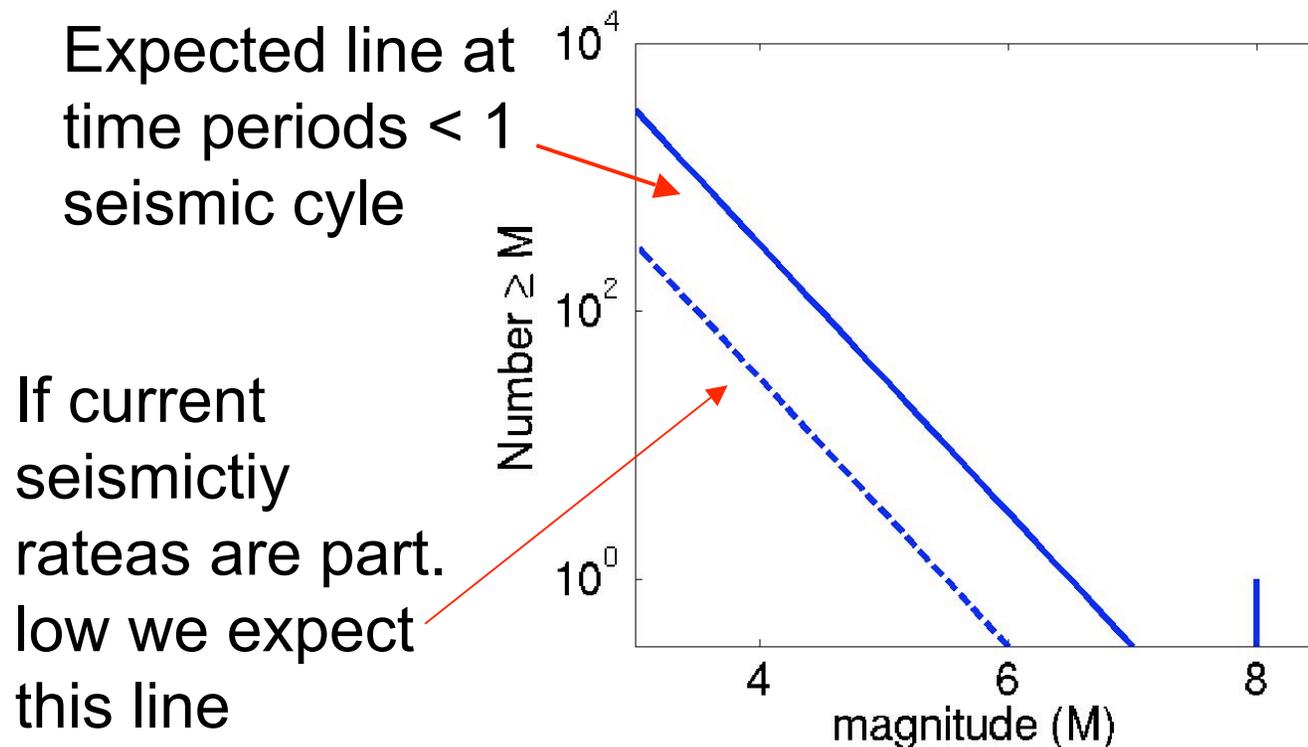
# 1) The Characteristic Earthquake Model

(Wesnousky *et al.* 1983)



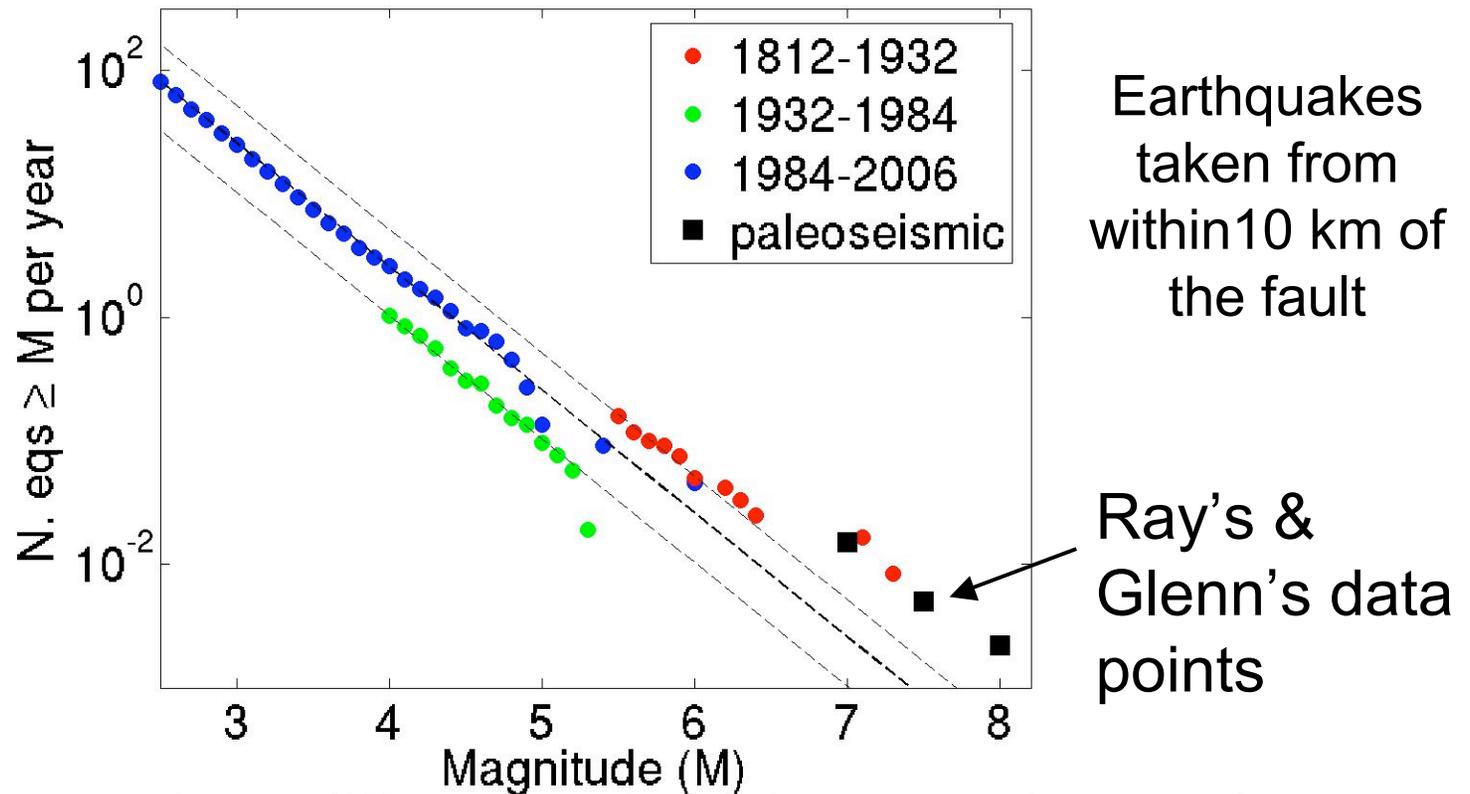
Observation claimed to show that large earthquakes on the fault are more probable than indicated by the GR relationship

Problem: Unknown total length of seismic cycle & long term seismicity rate. If underestimated, the small earthquake count cannot be expected to match



*Wesnousky* (1994) notes the California record not long enough to prove the Characteristic model

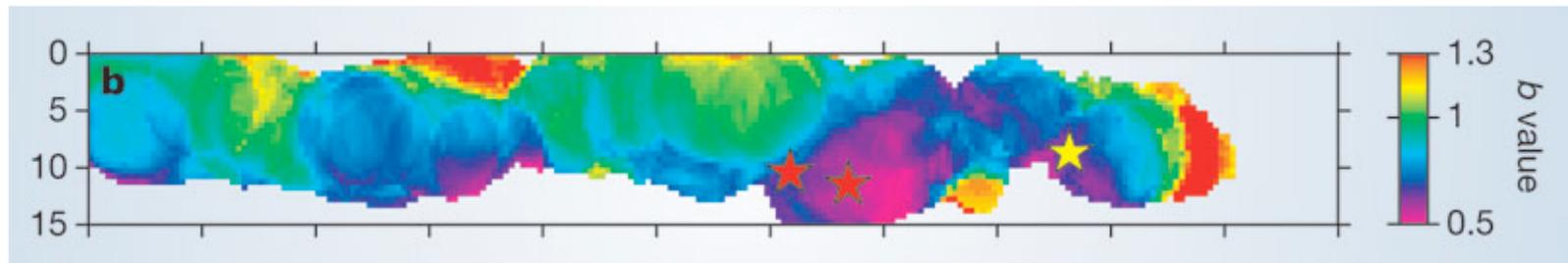
# Ray Weldon & I have been trying to compile the longest record possible for the Southern SAF



Different eras show different seismicity rates, but each era internally shows GR stats and agrees with the others within a factor of 2.

## 2) $b$ value varies with location

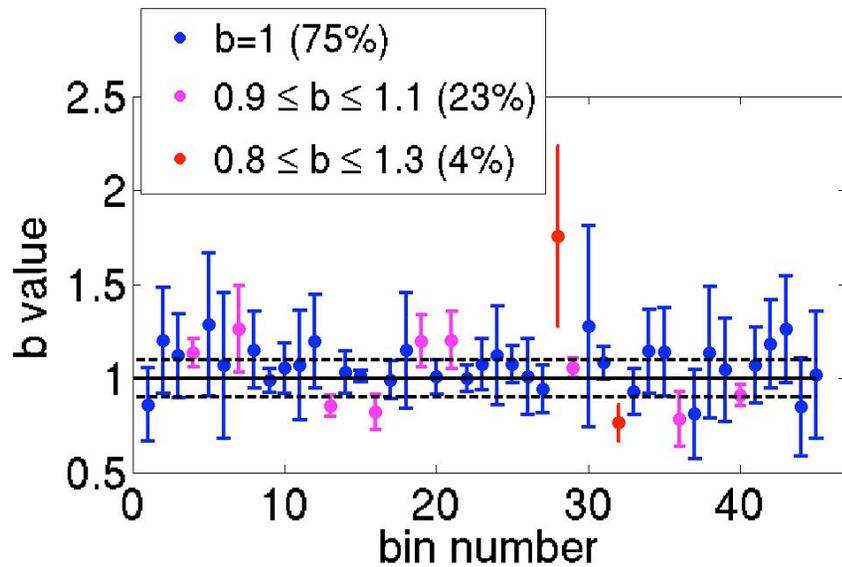
(*Shi and Bolt, 1982; Schorlemmer and Wiemer, 2004.....*)



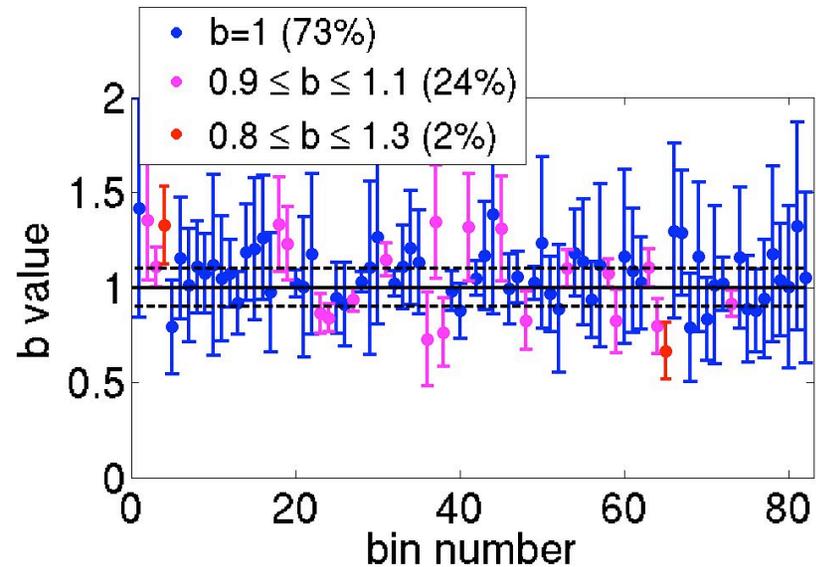
From *Schorlemmer and Wiemer, (2005)*

Problem:  $b$  value calculations naturally have high errors unless large ( $>2000$  eq) data sets with high magnitude accuracy are used

Dividing California into spatial bins of any size, most  $b$  values are consistent with  $b=1$  (or  $b=0.9$  to  $1.1$ ) within error



100 by 100 km bins



50 by 50 km bins

# Conclusions

- A simple model in which earthquake growth potential  $\propto$  current faulting area while resistance remains constant  $\Rightarrow$  universal GR relationship
- All data available for the Southern SAF indicates a GR magnitude distribution
- $b$  value variations seen in CA are in a narrow range when error is taken into account, supporting GR and  $b$  value universality

