The Wonderful World of Aftershocks

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Aftershocks are often seen as small, unimportant earthquakes

- The National Hazard Map and engineering codes ignore them!

- But aftershocks are important!
  * Majority of all earthquakes
  * Release tectonic stress
  * Amazing statistical properties
  * Mysterious physics….
  * The key to the propagation of tectonic earthquakes ?

Amazing aftershock statistic #1

Omori's Law: Aftershock rate decays in time as

$$R(t) = \frac{K}{(t + c)^p}$$

- Aftershocks can follow Omori's law for hundreds of years (*Ebel et al.,* 2000).
- Omori's law is empirical. No one has been able to derive it.

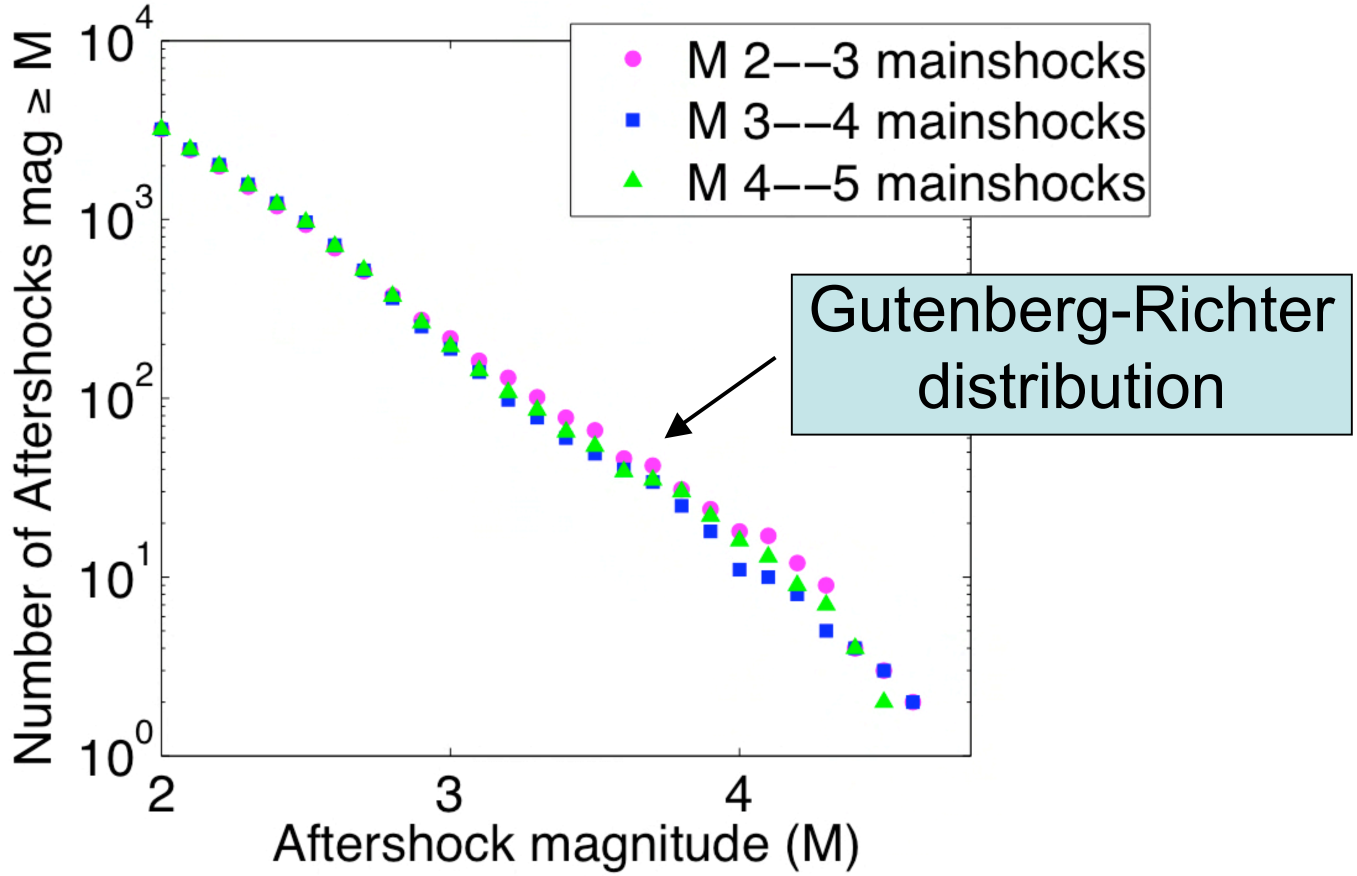
Amazing aftershock statistic #2

The magnitude of each individual aftershock is independent of the magnitude of its mainshock

[*Reasenberg and Jones* (1989), *Michael and Jones* (1998), *Felzer et al.* (2004)].

- Larger earthquakes have larger aftershocks only because they have more aftershocks.
- All foreshocks can be explained as earthquakes with aftershocks that are larger than themselves.

Evidence for Statistic #2: Aftershock magnitude distributions are independent of mainshock magnitude



Aftershocks taken from 2 days/5 km around each mainshock.
3200 aftershocks in each distribution

Foreshock rates can be accurately predicted from the rate of aftershocks smaller than the mainshock

1. No. aftershocks ≥ M = $F(M_{main})10^{-bM}$ ← GR distribution
2. $F(M_{main}) \sim 10^{Mmain - 1.3}$ in California, for aftershocks within 1 day and 1 fault length of mainshock; b=1.

Predicted rate that an earthquake will produce an aftershock larger than itself within one day:

$$=10^{Mmain-1.3-M} = 10^{Mmain-1.3-Mmain} = \mathbf{0.050}$$

Observed rate: 0.047 +- 0.0054

(6086 M ≥ 3 California earthquakes, 1984-2004)

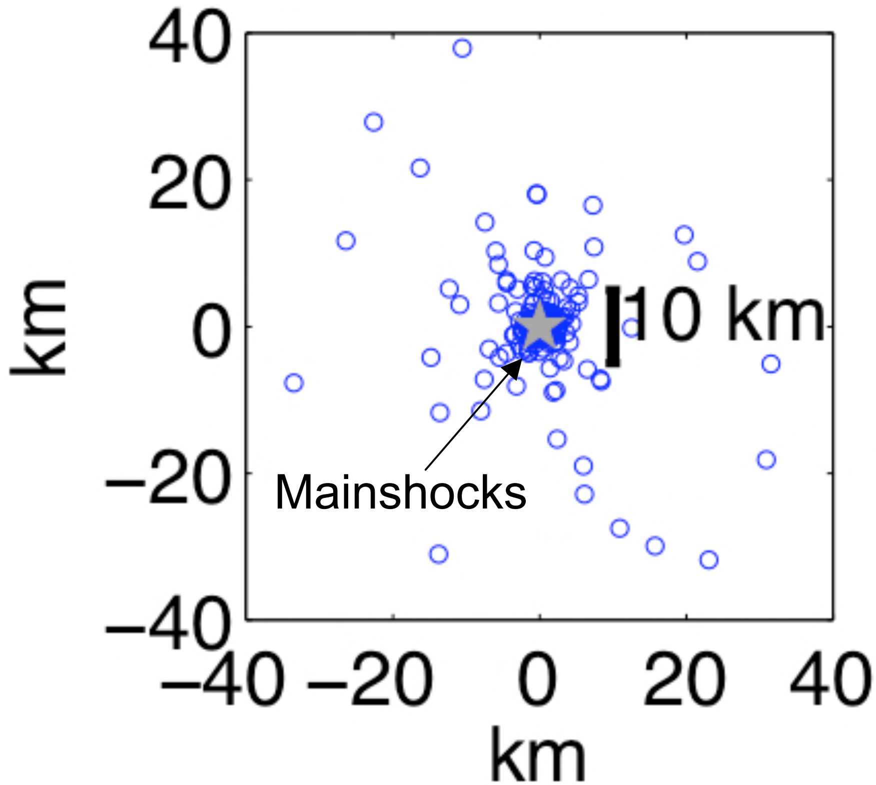
Amazing aftershock statistic #3

The distance of an aftershock from its mainshock is independent of the magnitude of the mainshock

[*Felzer and Brodsky, 2006*]

- **Distant aftershocks of small earthquakes are simply difficult to see because there are so few aftershocks/mainshock**
- The trick to seeing the distant aftershocks is to combine lots of aftershock sequences

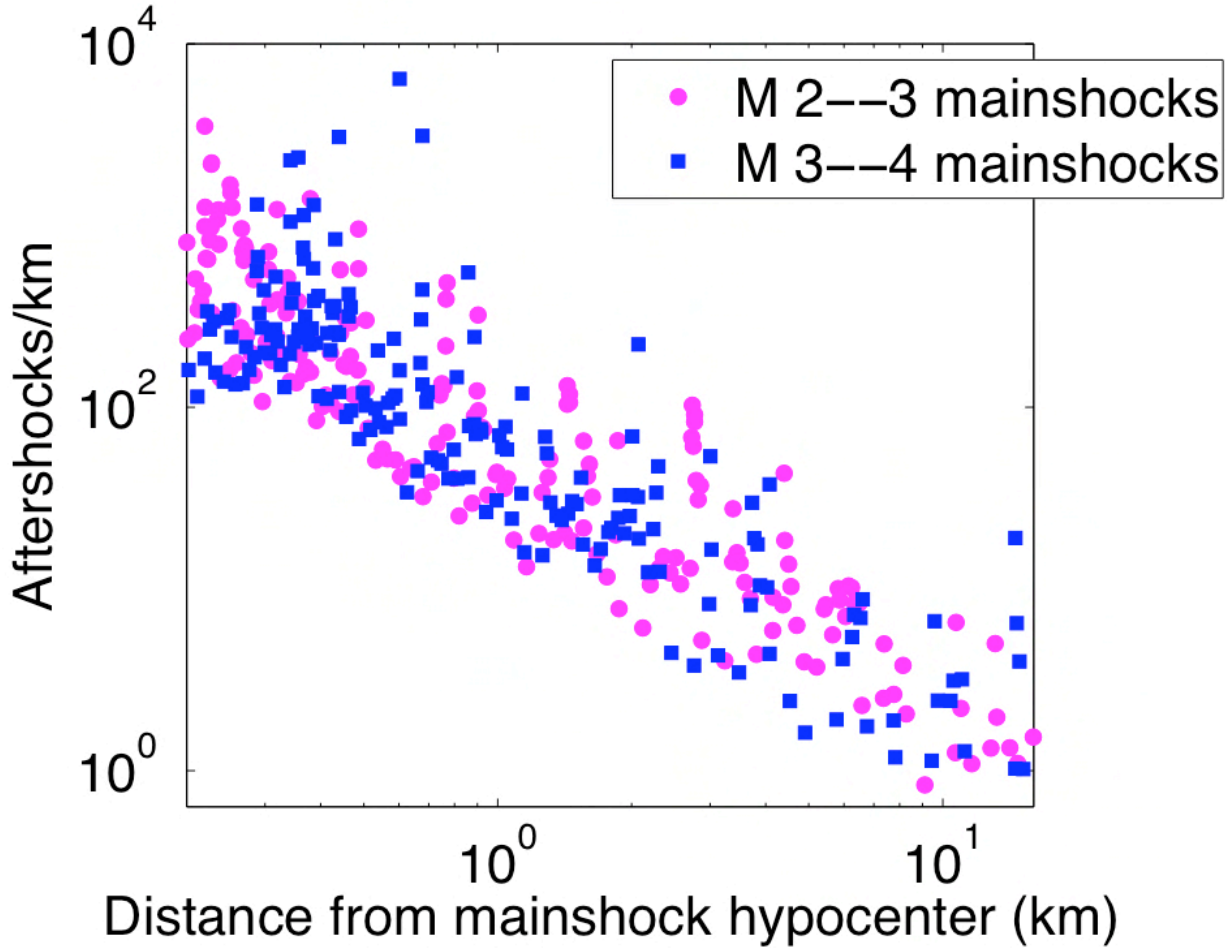
Spatial stack of the first 30 minutes of M≥2 aftershocks of 2,355 M 3--4 mainshocks



199 aftershocks plotted

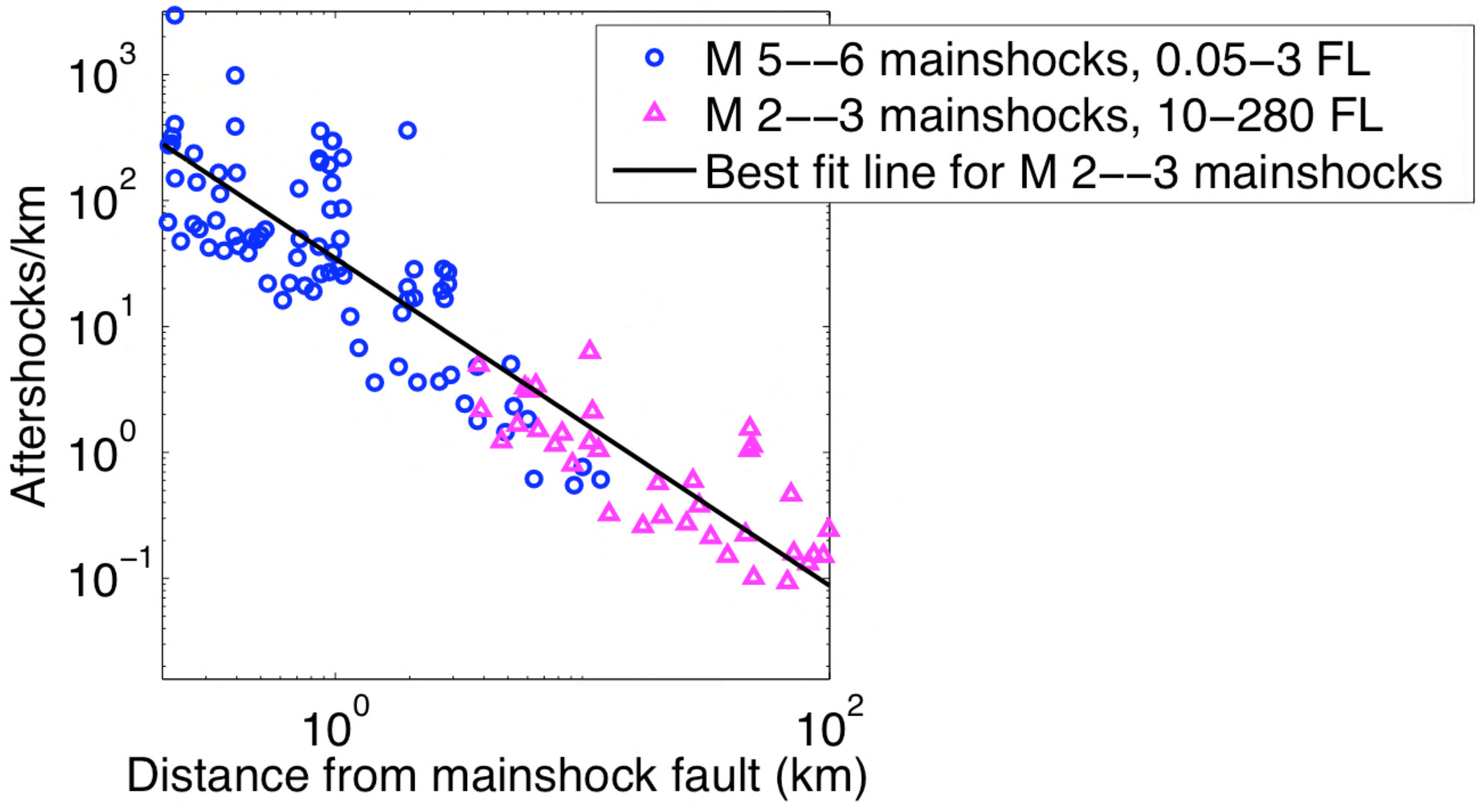
M 3.5 fault length ~ 0.6 km

The distribution of aftershock distances is independent of mainshock magnitude



Aftershocks taken from within 30 minutes of the mainshocks. 200 aftershocks in each data set

The rate of decay of aftershock density is also consistent over long distances



Aftershock density decay with distance is **constant from 0.05 to 280 Fault Lengths** => **Dynamic triggering throughout**

(*Felzer and Brodsky*, 2005)

Amazing aftershock statistic #4

The timing or ‘clock advance’ of an aftershock is independent of the amount of stress applied by the mainshock

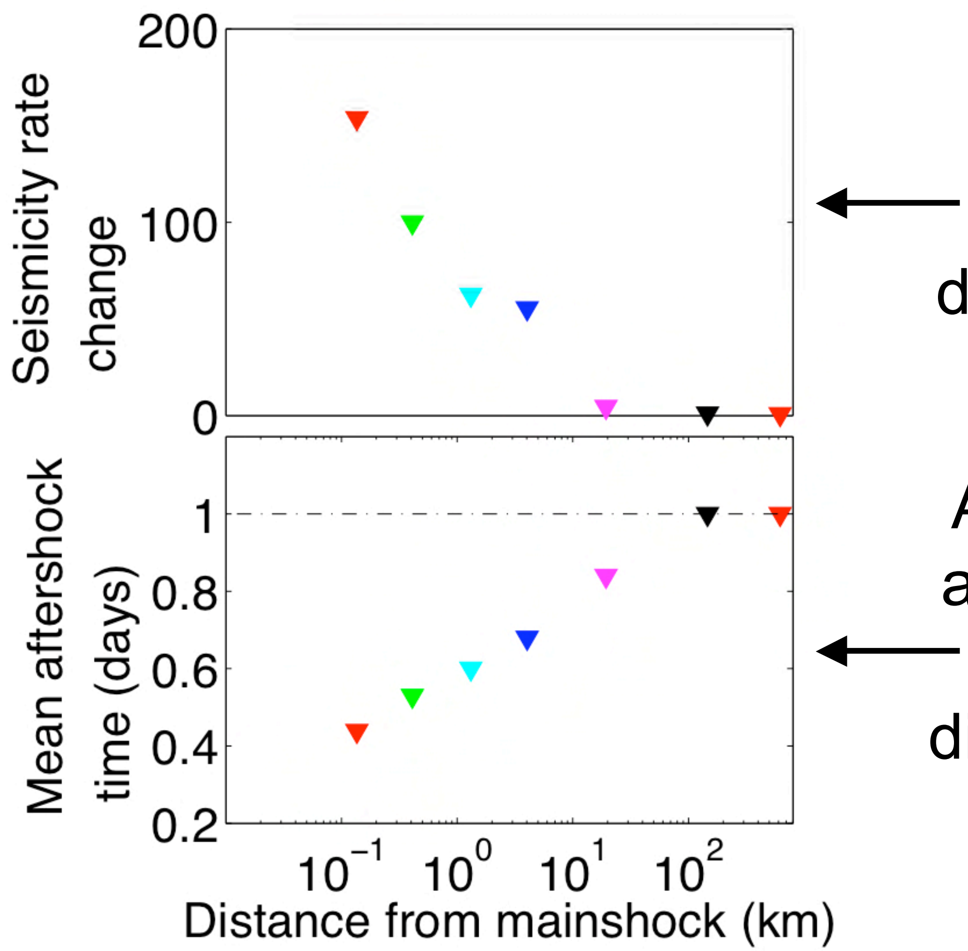
[Felzer 2005]

This observation negates most physical aftershock generation models, including the static stress change + rate and state friction of *Dieterich (1994)* !

In the static stress + rate and state model, aftershocks are earthquakes strongly time advanced by an amount proportional to the mainshock-applied stress

It’s like a mouse nudging clock hands -- the amount of clock advance ∝ amount of nudge

“Mouse nudge” model expectation: Lower stress change at larger distances ∝ later aftershocks



Aftershock density decreases with distance/stress change

And the mean time of aftershock occurrence increases with distance/stress change

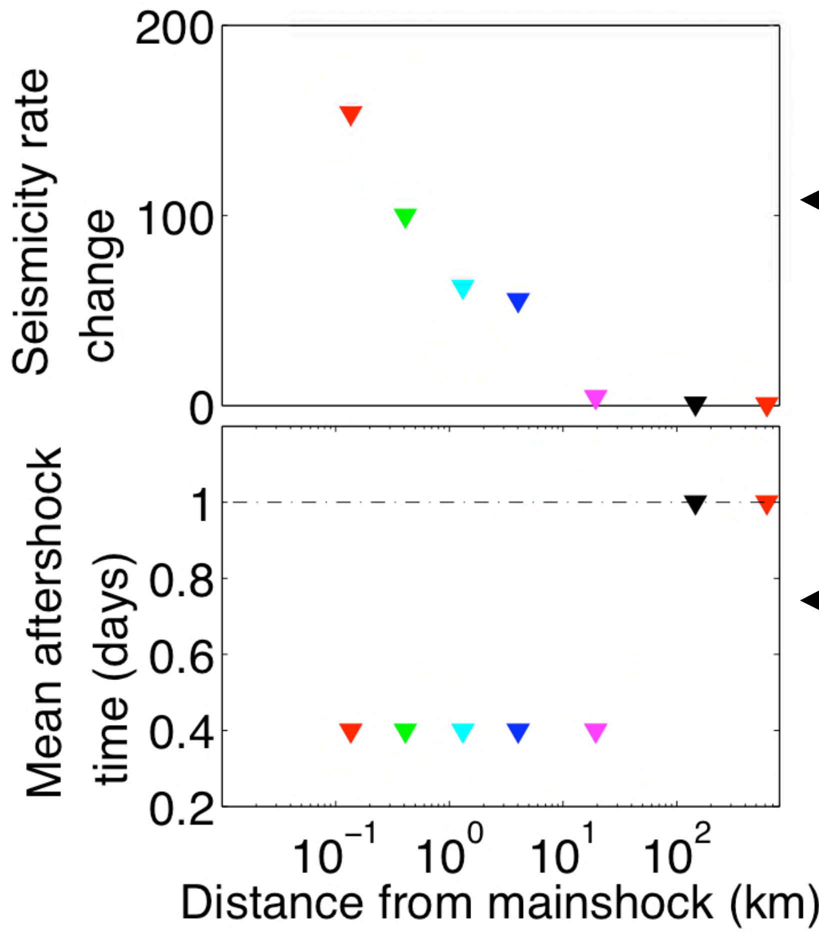
Alternative: The “Harold Lloyd” Model
Binary, not proportional, stress response

Option 1: Stiff clock or light Harold => no clock advance

Option 2: The hand gives way maximum clock advance

- If Harold is heavier, a clock advance is more likely
- But Harold’s weight ≠ clock advance size

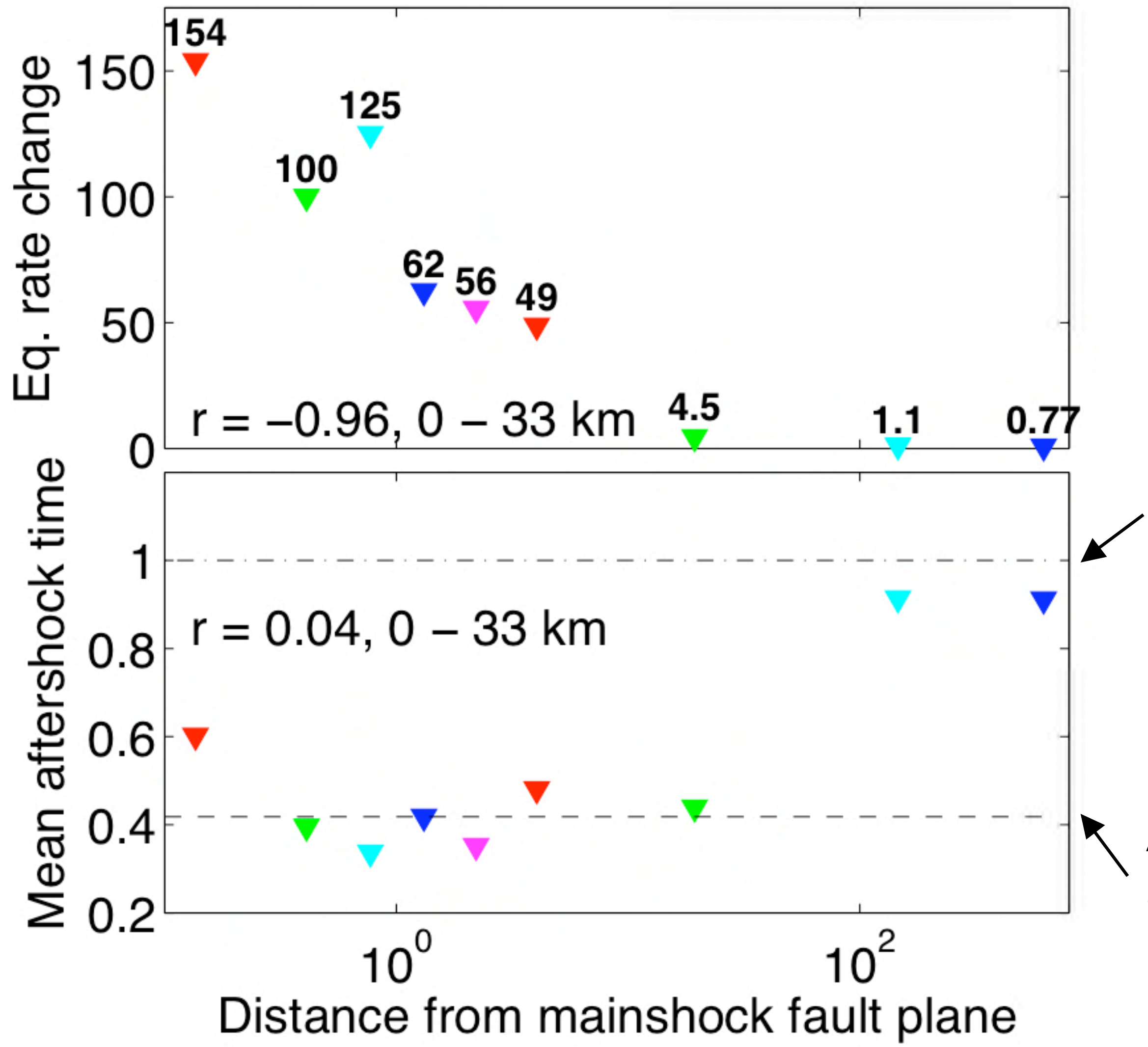
Harold Lloyd model expectation: Distribution of aftershock times is the same at all distances



Aftershock density still decreases with distance/stress

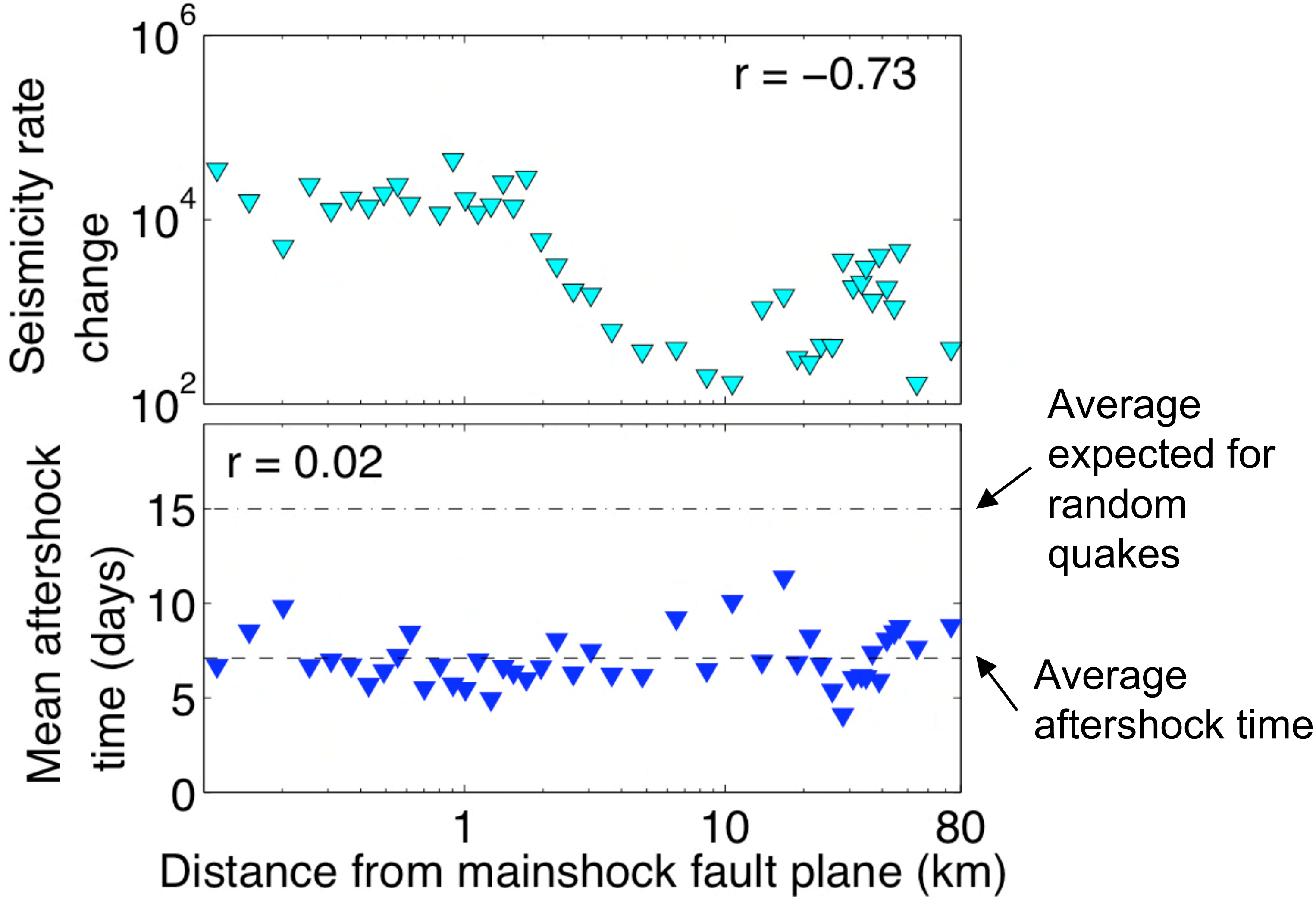
But the mean time of aftershock occurrence doesn't change with distance/stress

Data: Mean aftershock time does not vary with stress change



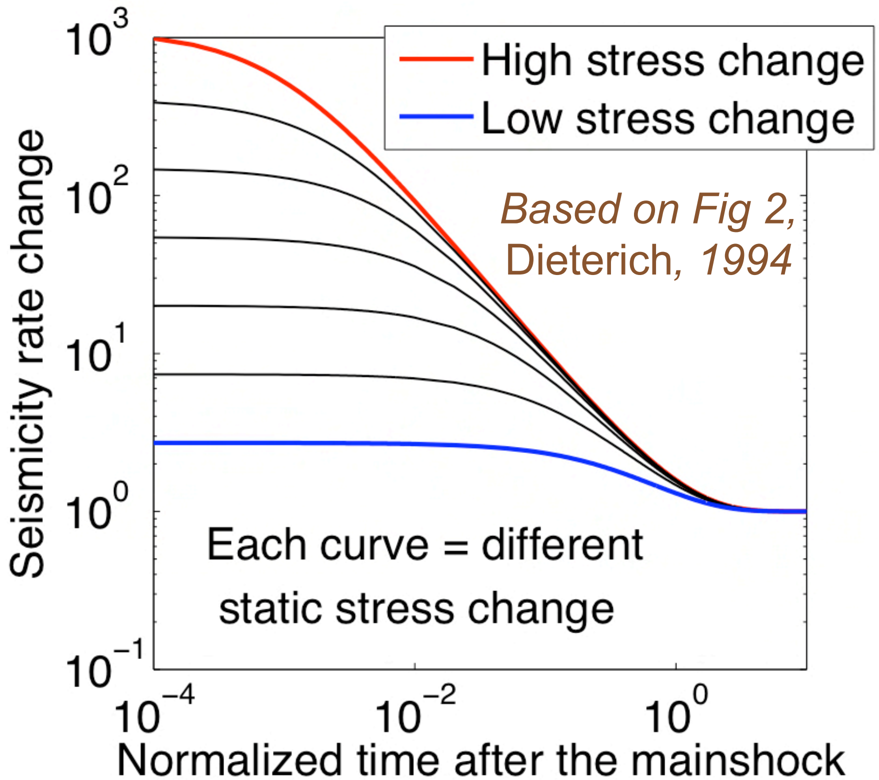
Aftershocks measured over first 2 days of sequences, M 5-6 mainshocks

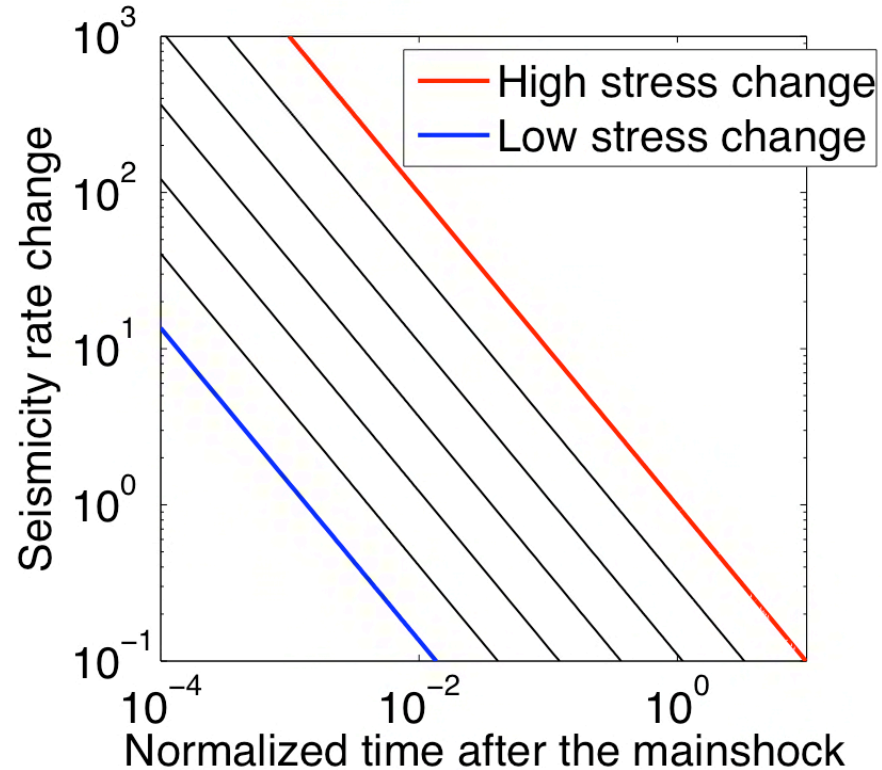
M 7.3 Landers earthquake: Average aftershock time is independent of stress change



Aftershocks measured over first 30 days of sequence

Another test of Harold vs. the mouse: Inspect full seismicity decay rate





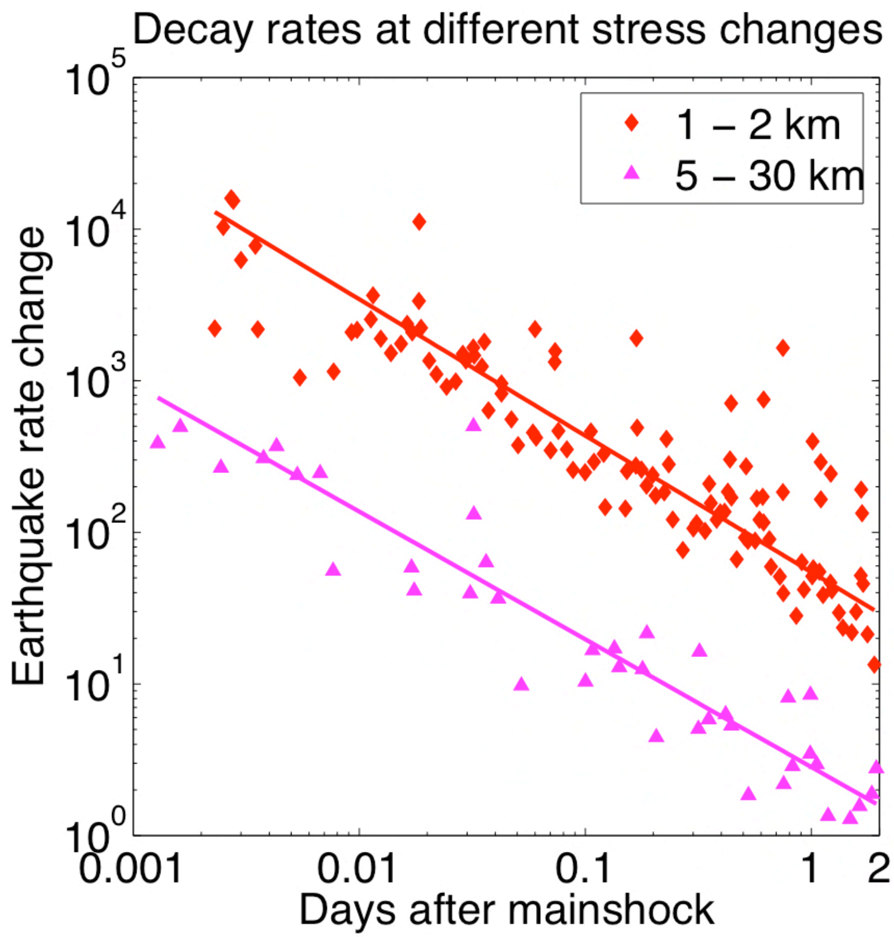
"Mouse nudge" prediction

Fewer rapid aftershocks at lower stress: Omori Law *c* value increases with distance

"Harold Lloyd" prediction

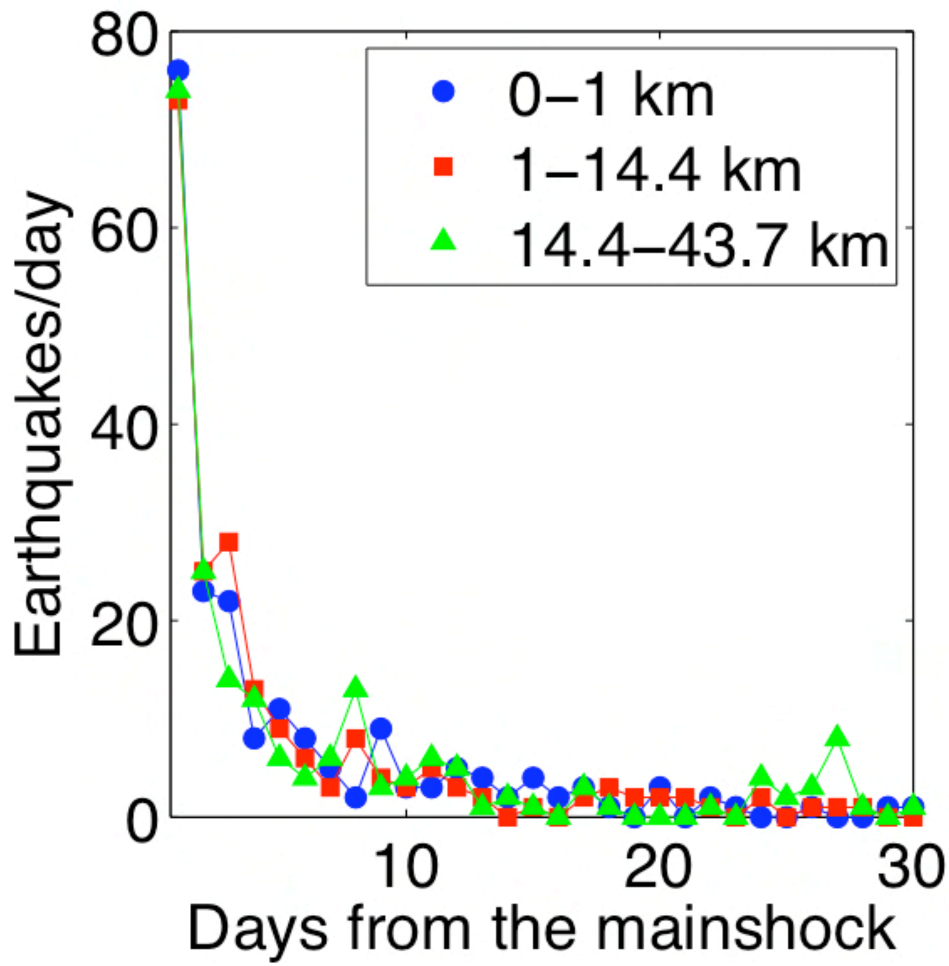
Decay rate the same at all stress changes

M 5 - 6 mainshocks: Seismicity rate change decay does not depend on distance



Agrees with the results of *Jones and Hauksson* (1998) that the value of *c* is constant with distance

Landers mainshock: Groups of 200 aftershocks at different distances show the same decay rate



Kolmogorov Smirnoff Test: All distributions similar at 95% confidence

Result: The correct physical aftershock model must contain binary, on/off aftershock triggering

- Many stochastic aftershock models (*Kagan* (1982), *Ogata* (1998), and others) use binary triggering.
- But I am not aware of a physical aftershock model that satisfies binary triggering ?
- New physical models needed!

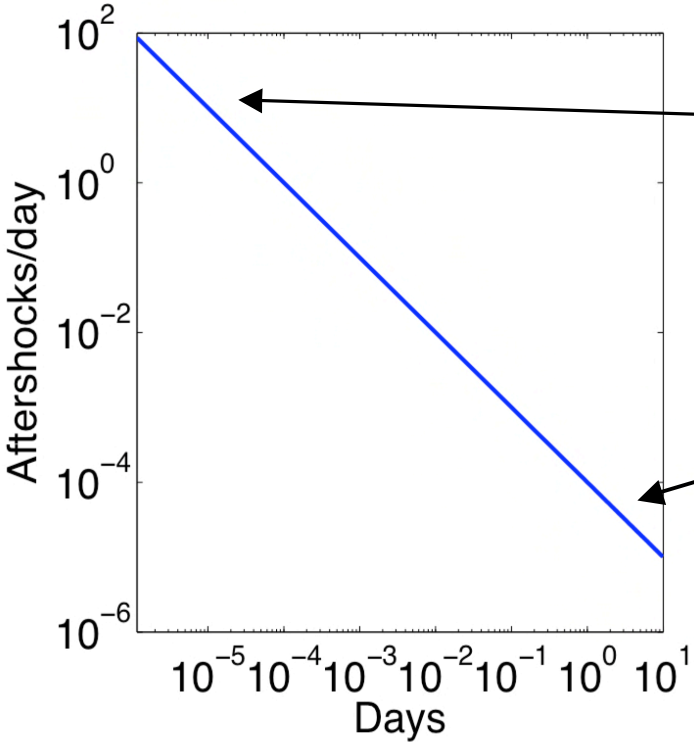
Amazing aftershock statistic #5

Static stress changes appear inefficient at triggering aftershocks

- *Pollitz and Johnston* (2006) found that for similar earthquake and aseismic events, the earthquakes produced many more aftershocks.
- Reliable stress shadows cannot be found (*Marsan*, 2003; *Mallman and Parsons* 2004; *Felzer and Brodsky* 2005).
- Static stress change patterns improve aftershock location predictions by ~8% (*Parsons*, 2002). Dynamic stress patterns do at least as well (Kilb et al. 2000).

Amazing aftershock statistic #6

If you project Omori's law back far enough, you can make a mainshock!



If the time gap between events comes from here, we see a single, larger earthquake

If the time gap comes from here we see a separate mainshock and aftershock

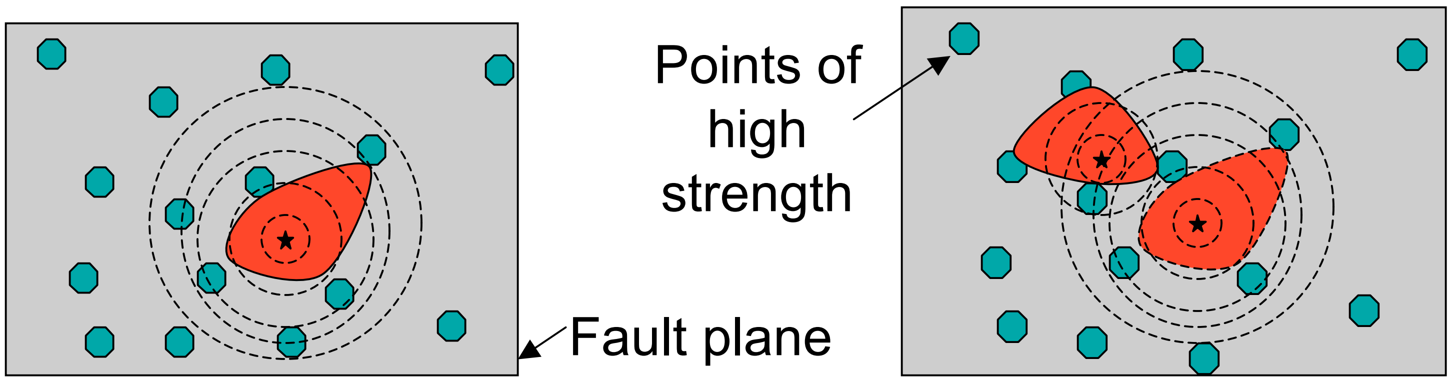
Kagan and Knopoff **(1981)**

Statistic #6 suggests that mainshock are avalanches of subevent aftershocks. Subevent earthquake models are common

- *Housner* (1955)
- *Vere-Jones* (1976)
- *Kagan and Knopoff* (1981)
- *Zeng et al*. (1994)
- *Rydelek and Sacks* (1996)
- *Lavallee and Archuleta* (2003)
- …….

The innovation is to connect the "avalanche" & aftershock processes

Conceptual Model



Earthquake remains small **OR** An aftershock subevent is triggered; earthquake grows

- A subevent nucleates at a point of weakness
- The subevent is stopped by strength heterogeneities when it reaches area *A* unless its co-seismic stress can trigger additional subevents, via aftershock-trig physics.
- Each subevent has the same average area (controlled by the heterogeneity) and potential to trigger others

Preliminary stochastic Monte Carlo modeling shows that the aftershock avalanche model can produce earthquakes with Gutenberg-Richter magnitude-frequency stats and realistic time series

Monte Carlo Simulation Basics

- **Model starts with a few random subevents**
- **Locations of triggered subevents:**

  Chosen randomly from distribution $cr^{-1.4}$

  (Felzer and Brodsky, 2006)
- **Time of triggered subevents:**

  S wave travel time

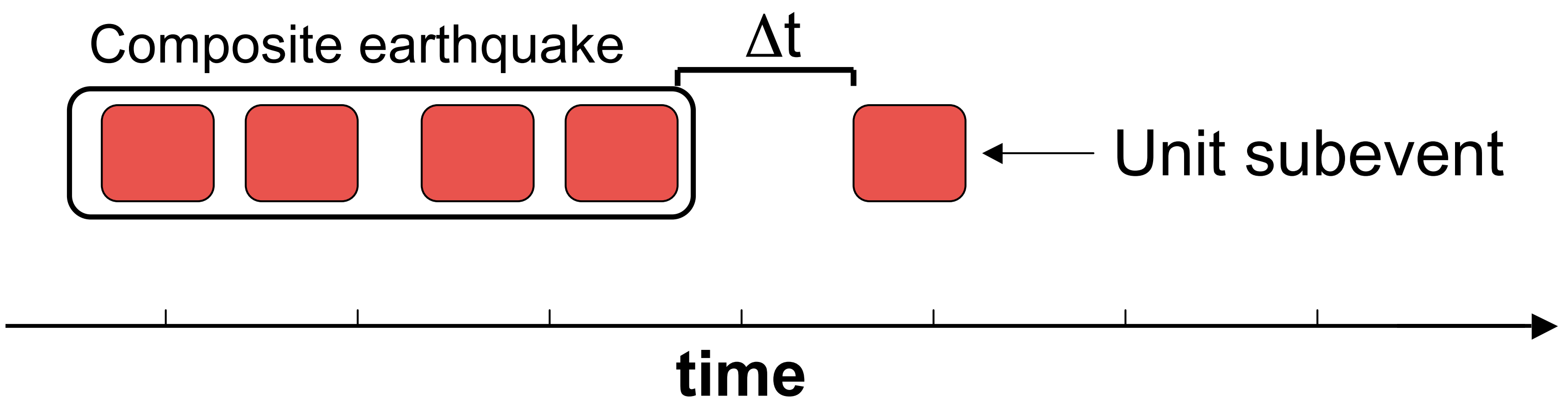
  + minimum wait time (t_{min})

  + Omori's Law (at^{-p})
- **Subevent triggering is binary. Timing is not dependent on stress change, and events can only be triggered once.**

Monte Carlo Simulation Parameters

Parameter	Value	Reason
p	1.34	*Felzer et al*. (2003)
a	0.0013	Reproduces Båth's Law for > 20 s bet. eqs
t_{min}	0.007 seconds	Each subevent produces ~ 1 aftershock

Monte Carlo Simulation: Combining subevents into earthquakes

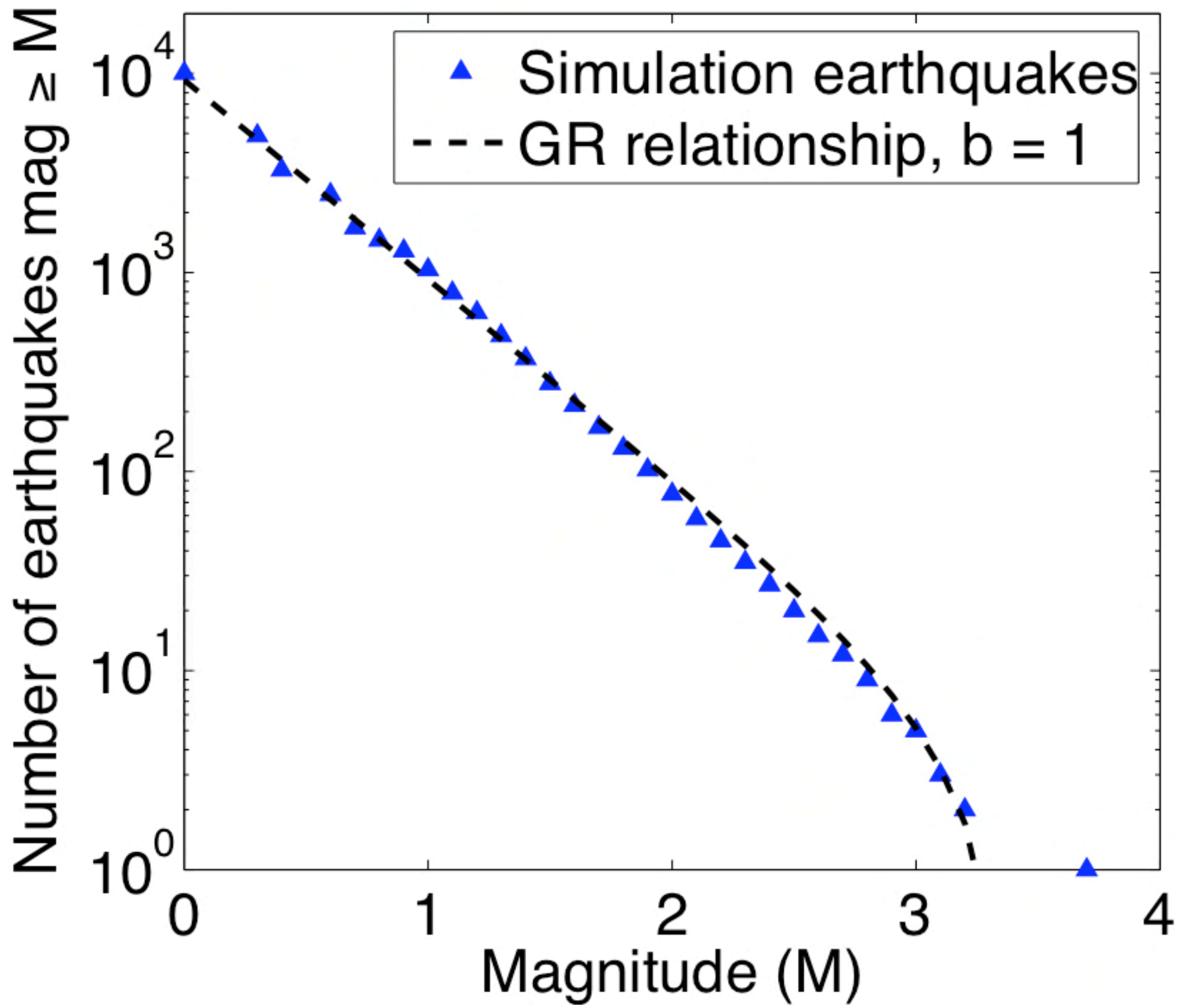


fudge factor

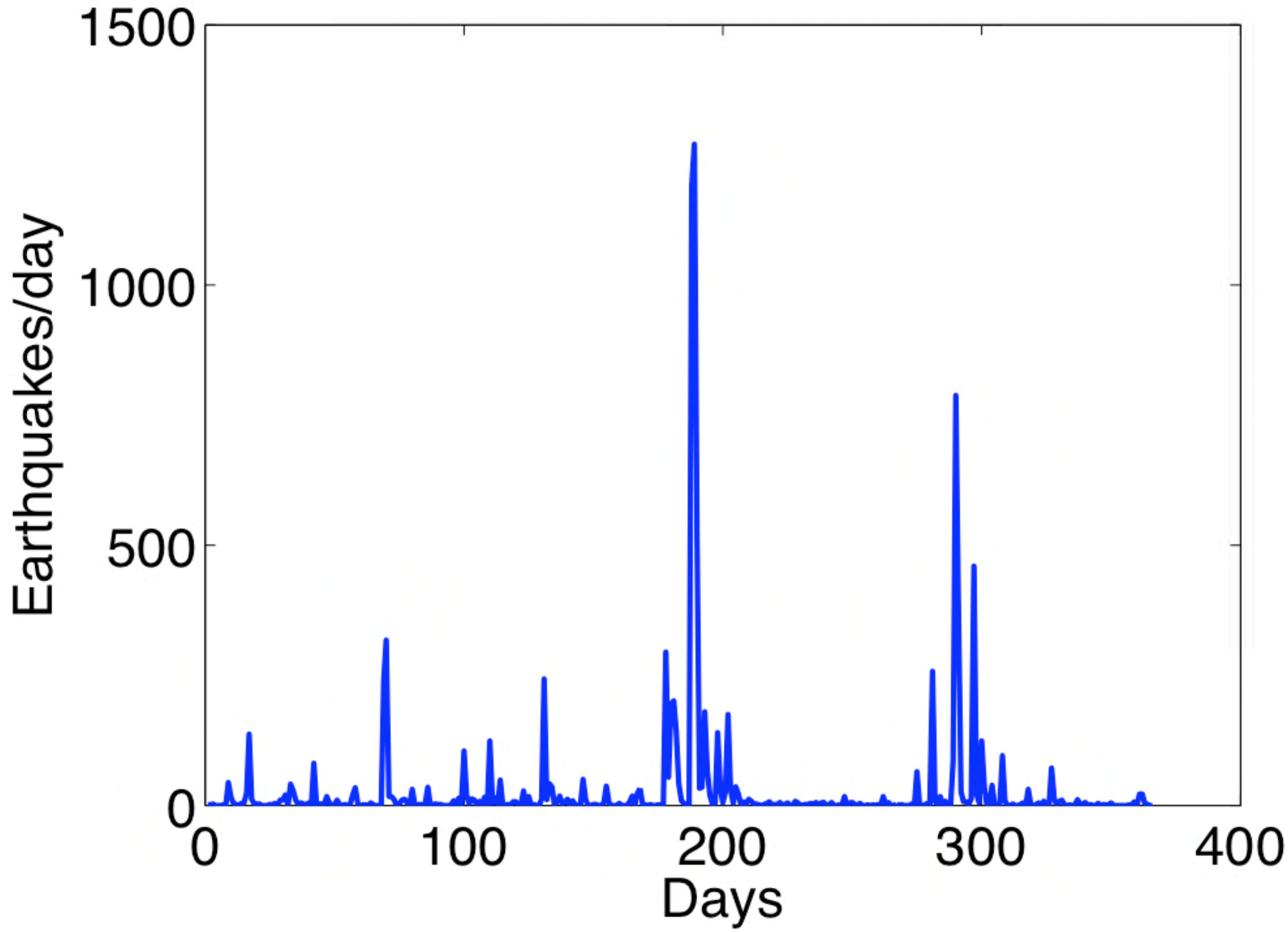
Proxy for earthquake duration

- If $\Delta t < (\mathbf{0.2})(\sqrt{\text{no. subevents in existing earthquake}})$ => Subevent is added to the preceding earthquake
- Composite Magnitude= log(Earthquake Area)

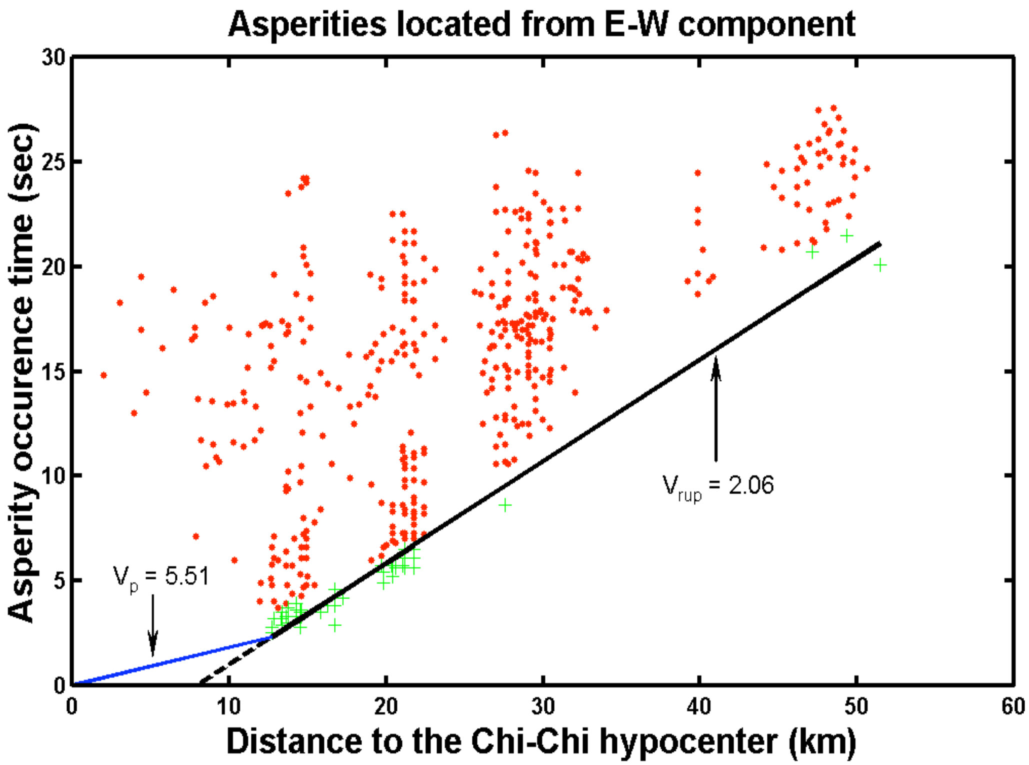
Simulation results: G-R relationship, *b* = 1

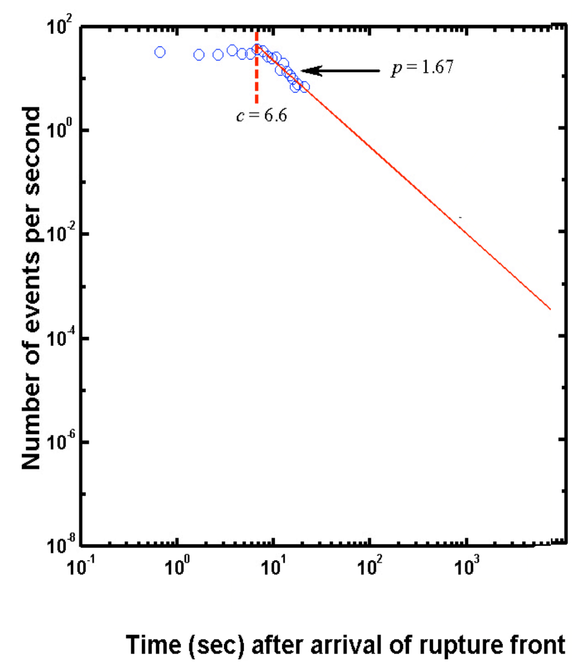


Simulation results: Time series of coalesced earthquakes



Direct observation of the proposed subevents: High frequency bursts following Omori's Law during Chi-Chi





Chen et al. (2006) and *Fischer and Sammis* (2005)

The aftershock avalanche model agrees with our observed aftershock statistics

Observed:

Aftershock magnitude, location, and timing are independent of mainshock magnitude

Model:

Predicts observation. Aftershock triggering is accomplished by the initiation of tiny subevents that are the same in all earthquakes => all aftershock properties should be independent of mainshock magnitude.

Agreement between model and observed aftershock statistics

Observed:

The time an aftershock occurs is independent of the amount of stress applied.

Model:

This observation is one of the rules in the model. Without it, earthquakes would accelerate and scale invariance would not be preserved.

Agreement between model and observed aftershock statistics

Observed:

Most or all aftershocks are most likely triggered by dynamic stress changes

Model:

Aftershock triggering and mainshock propagation are the same thing, and mainshock propagation is understood to be a dynamic process.

Conclusions

- Aftershock magnitude, distance, and timing are independent of mainshock magnitude.
- Aftershock timing is independent of mainshock-induced stress => aftershocks cannot be triggered by static stress change + rate and state friction.
- The idea that earthquake propagation = aftershock triggering is consistent with earthquake & aftershock statistics.